Effect of a critical field on screened dielectric-breakdown growth

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In this paper, we have shown that, due to a critical field, the patterns growing in the screened Laplacian field will be affected. When the critical field is zero, there is a transition from a dense growing to a single branch of the aggregate. At a higher critical field, the pattern shows a spiky-type aggregate and for a small critical field, the transition is cut. For an intermediate critical field, the result shows that, below the transition, there is no dense structure.

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I. INTRODUCTION

Much of the recent interest in fractal-growth phenomena has arisen because of the universal nature of the observed patterns. The diffusion-limited-aggregation (DLA) and dielectric-breakdown (DB) models have been very successful in illustrating the possibility of fractal growth in a Laplacian field [1,2]. The growth of metallic aggregates through electrochemical deposition (ECD) either has a fractal character, as in diffusion-limited aggregation, or yields dendritic crystals, or gives rise to dense radial structures [3–5]. In electrochemical deposition, a transition from a dense pattern to a more diluted branched structure has been observed and referred to as the Hecker transition. More recently, a study of the effects of screening on structures growing in electrostatic fields was reported where the Laplacian equation has been replaced by

$$\nabla^2\phi=\lambda^2\phi \ , \tag{1}$$

the linearized Poisson-Boltzmann equation [6]. The origin of screening might lie in the presence of free charges, such as in the cases of electrochemical deposition and dielectric breakdown. The results show that screening leads to a much richer variety of patterns. It introduces a new length scale and a nontrivial dependence on the boundary conditions which is responsible for a transition that resembles the Hecker transition: a pattern may have a fractal character at scales shorter than the screening length, be Eden-like (i.e., compact), or grow dense, and then it follows a transition from dense to single-branch growth. This transition is characterized by a change in the sign of the electrostatic field at the aggregate [6].

However, in our opinion, the transition found in Ref. [6] will be affected by the existence of a critical field in the growth of the pattern. Our aim in the present paper is to study the effect of the critical field on the pattern growth. Actually, the dielectric-breakdown model was generalized to include lower cutoffs, which prevent growth at low field; for example, one cutoff is at the critical fields [7]. It is shown that the growth will stop after a crossover from the usual DB patterns to a spiky behavior [7,8]. Another motivation of the introduction of the cutoffs is to model a capillary pressure into the problem of viscous fingering. If the displacement is of a wetting fluid by a nonwetting fluid, then the capillary pressure prevents the nonwetting phase from entering a pore when the pressure is lower than the local capillary pressure P_c [9]. In simulations of porous media as a network of ducts, P_c depends only on the ducts' width via the Laplace law. Taking the radii of the ducts to be constant all over the network, we conclude that each bond will be active if the viscous pressure on it, P, is bigger than P_c, and will not be active if $P<P_c$. We see that the capillary pressure is equivalent of the critical field [8]. In addition, in the solidification the kinetic effects (undercooling and surface tension) also in some ways, similar to the case mentioned above, are singular perturbations affecting the pattern formation.

In this paper, we first follow the standard growth procedure of the dielectric-breakdown model [2] and numerically solve Eq. (1) by using an iterative method. Then we grow the patterns, using numerical simulations in the presence of four different critical fields in a planar geometry and present a discussion.

II. RESULTS AND DISCUSSION

For the numerical simulations, we use a planar geometry with a 100×200 square lattice and the boundary conditions $\phi^i=5\times10^{-7}$ on the aggregate and $\phi^o=1$ on the fixed boundary, as well as a periodic boundary condition on the x direction. The aggregation is related to the local electric field E_j, i.e., with probability

$$P_j=\frac{|E_j|}{\sum_j |E_j|} \ , \tag{2}$$

where $E_j=-\nabla\phi$ and ϕ is obtained by iterating the screening equation (1) with $1/\lambda=10$ being the screening length. The effect of the critical field E_c is introduced in Eq. (2). This responds to a cutoff on the gradients, i.e., there is no growth at point j if $|E_j|\le E_c$, and the surviving probabilities are renormalized, $P_i\to P_i/P_s$, where $P_s=\sum_k P_k$ for those k's for which $|E_k|>E_c$.

In Fig. 1 we show the patterns with four different criti-

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cal fields, $E_c=0$, 4×10^{-4}, 5×10^{-9}, and 5×10^{-8}. We can see that for zero critical field, $E_c=0$, there is a transition. The growth is divided into two stages: (I) At the beginning, the growth is just like the dielectric-breakdown model; however, due to the screening, the growth before the transition becomes denser. Almost all the perimeters are grown, and this gives rise to a dense or Eden-like pattern. (II) Because of the instability for all wavelengths just beyond the transition, once a small tip develops it will be amplified and the pattern appears like a branch of a tree. At the same time, the electric field at the surface of the aggregate, $E(l)$, changes its sign during

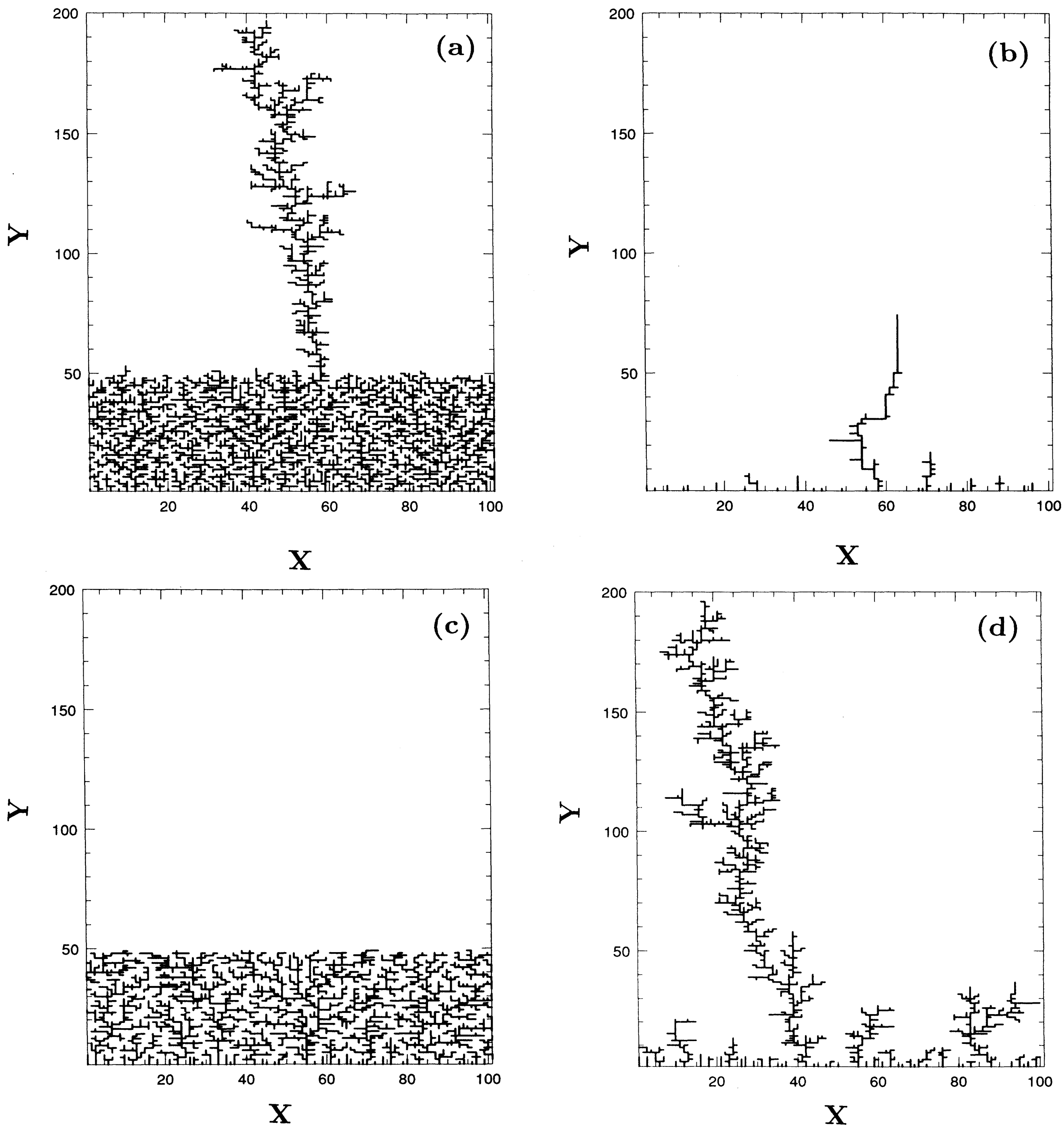

FIG. 1. Dielectric-breakdown patterns of the screened Laplacian growth with critical field. (a) $E_c=0$; (b) $E_c=4\times10^{-4}$; (c) $E_c=5\times10^{-9}$; (d) $E_c=5\times10^{-8}$.

the transition, and it vanishes at the transition. For convenience we can define an electric-field value E_t near the transition which is actually small, and we discuss the cases for $E_c \gg E_t$, $E_c > E_t$, and $E_c \sim E_t$, respectively. We can expect the situation to be different from that of $E_c = 0$. The whole growth process for the pattern is such that, before transition, the growth occurs for those perimeters for which the probabilities are larger than that of the tip, which may develop into a single branch of a tree. If the cutoff is on these perimeters, the growth of the pattern may completely stop, and if the cutoff is just around the transition there will be no transition.

When $E_c \gg E_t$, $E(l)$ may not reach the value E_t, since the growth is cut off much earlier and so without perimeters with $|E_j| > E_c$, and the growing is stopped before the transition takes place. This is just the case shown in Fig. 1(b), with $E_c = 4\times10^{-4} \gg E_t$. The pattern is a spiky-type structure [8]. Nevertheless, for the case of $E_c \sim E_t$, since the cutoff is much smaller and just around the transition, almost all perimeters are grown, and so the pattern appears very dense. No sharp tip can develop. Thus there is no transition [see Fig. 1(c)]. However, the interpretation for the case of $E_c \geq E_t$ is slightly different. Below the transition, some perimeters are cut, and some are not. The uncut perimeters will grow. After the rearrangement of the potential, unlike the case for Fig. 1(c), due to the dilute pattern structure, for few perimeters, the gradient of the potential or the electric field may be larger than the critical field E_c, and these perimeters grow. The unstable tip is amplified to give rise to a tree structure [see Fig. 1(d)]. Actually, the discussion above can be drawn from the ratio between the instantaneous rates of growth of the perturbation (δ) and that of the flat surface (l), which is (see Ref. [6])

$$\alpha_m = \frac{\dot{\delta}/\delta}{\dot{l}/l} = \left| (\lambda^2+m^2)^{1/2} - \frac{\lambda^2\phi^i}{E(l)} \right| l \ . \quad (3)$$

Because of the existence of the cutoff, the existence of a critical field, $E(l)$ cannot become zero (at the transition), instead of a finite value E_c. Thus the growth rate α_m for all m does not go to infinity and the transition may not appear as sharp as Fig. 1(a).

In Fig. 2 we have plotted the average growth speeds, the averages of absolute values of the fields at the aggregate surface. There are minimal values for the velocity and the minimum corresponds to the transition. For curve 1, which corresponds to Fig. 1(b), the velocity is a constant, and the pattern is a spiky-type and is stopped by such a large critical $E_c = 4\times10^{-4}$. Curve 2 corresponds to Fig. 1(d). The pattern below the transition is the same as the dielectric-breakdown growth. However, one tip has developed into a single tree. Curve 3 is for smaller E_c; the cutoff is just about the transition [see Fig. 1(c)]. Curve 4 describes the case of zero critical field [Fig. 1(a)].

The position of the transition depends on the boundary conditions of ϕ^i, ϕ^o, the screening length λ^{-1}, and the system size L, and it is very sensitive to the values of ϕ^i and ϕ^o for the fixed λ and L. Since in our four cases the values of ϕ^i and ϕ^o are kept constant, the relevant singularities happen at the same position $Y = 50$ in all the figures. Similarly, for different values of ϕ^i and ϕ^o, although the positions of transitions are shifted, the physical relevances are the same. There is always an electric field for the transition E_t and the structures of the patterns are only affected by the value of the critical field E_c. We checked this by some other iterations. Actually, this appealing feature is for the case of $\phi^o > \phi^i$ because of the electric field $E(l)$ changing its sign at a position which is related to the transition. Nevertheless, for the case of $\phi^o < \phi^i$, it has been shown that there is no transition since the field at the surface of the aggregate has the same polarization of the electrodes in the whole growth [6].

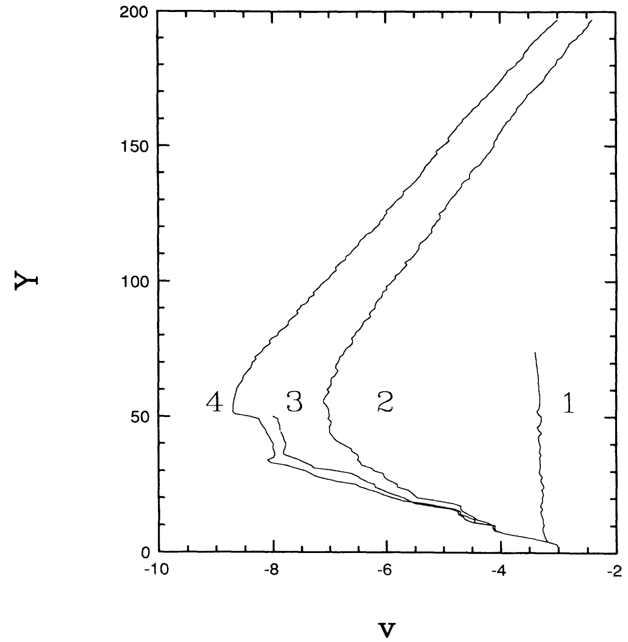

FIG. 2. The growing velocity: the average of the absolute value of electric field at the surface of the aggregate $E(l)$. The logarithm of $E(l)$, $\log_{10}(|E(l)|)$ vs Y. Curve (1): $E_c = 4\times10^{-4}$; Curve (2): $E_c = 5\times10^{-8}$; Curve (3): $E_c = 5\times10^{-9}$; Curve (4): $E_c = 0$.

III. SUMMARY

In this work, we have shown the effects of the critical field on the screened dielectric-breakdown pattern. The existence of the critical field does indeed affect the structures of the patterns. For a higher critical field E_c (or higher cutoff of the growing probability), the pattern shows a spiky-type aggregate. For a small critical field, the transition is cut. For the intermediate critical field, the structure of the pattern shows that below the transitions there is no longer dense growth.

The essential idealization made in the original dielectric breakdown model is the assumption of a perfect conducting phase growing in the perfect insulator and the omission of a critical field $E_c = 0$. The latter assumption is equivalent to the statement that the process is independent of the applied voltage. From a physical point of view this assumption seems to be a bit too strong [7].

The other aspect of interest of our study is that, in the practical case, $E(l)$ will never be zero due to the existence of the fluctuations. This situation responds to the existence of the cutoff E_c in the model. When E_c is small, the growth process may be such that, below the transition, there may not exist a denser structure. This is the real case in electrochemical-deposition aggregates, in which, before the Hecker transition, the pattern is less dense [3–5].

Finally, we add that an extension of the problem in question has been reported based on the biharmonic equation [10].

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